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Polarization Singularities in Partially Coherent Combined Beams

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Vector singularities are predicted and discovered experimentally in partially polarized combined fields formed by incoherent superposition of orthogonally polarized beams. Such singularities are *U* contours with zero degree of polarization and isolated *P* points with unit degree of polarization centered at vortices of the orthogonally polarized component of the combined beam. Crossing a *U* contour switches the polarization state to the orthogonal one. The above-mentioned singularities are adequately described in terms of the complex degree of polarization in the Stokes-space representation. It is shown that the field elements corresponding to the extrema of the complex degree of polarization form the vector skeleton of a partially coherent nonuniformly polarized field.

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C points at which the polarization azimuth of a circularly polarized field is indeterminate and *L* lines along which the azimuth of linear polarization varies smoothly and the direction of electric-field rotation is indeterminate are key objects in the singular optics of strictly coherent nonuniformly polarized fields resulting from, e.g., the stationary multiple scattering of laser radiation. The set of *C* points and *L* lines obeys the known sign principles and constitutes the vector skeleton of a coherent polarization-nonuniform field [1–4], i.e., the qualitative behavior of the field, in particular, the variation in its polarization, can be forecasted using the field parameters at its *C* points and on its *L* lines. This feature of the polarization singularities called *genericity* in [5] follows from their structural stability under small perturbations of the initial conditions or perturbations of a propagated beam. In addition, *nongeneric* polarization singularities formed due to the incoherent mixing of weighted Laguerre–Gaussian (LG) modes with orthogonal polarizations and different radial indices were considered in [6]. Universal theoretical and experimental methods for studying vector singularities involve the determination of the Stokes parameters as functions of the spatial coordinates in the studied cross section of the field, the subsequent determination of the spatial distributions of the polarization azimuth and ellipticity, and the separation of the singular elements of the field.

A new problem arises if at least one of two *incoherently* mixed orthogonally polarized beams contains phase singularities (optical vortices [7]), i.e., points in the beam cross section at which the field amplitude is zero and the phase is indeterminate and changes stepwise by π upon crossing such a field point. The known singularities such as optical vortices, *C* points, and *L* lines are absent in the combined beam formed by the incoherent coaxial superposition of the above-mentioned beam with an orthogonally polarized plane wave or another vortex beam with an orthogonal polarization. The states formed at almost any point of the cross section of the combined beam are "mixed" in the sense of the quantum mechanics or statistical electrodynamics [8, 9], i.e., correspond to a partially polarized combined beam whose degree of polarization is generally a function of the spatial coordinates. The states of the field are "pure" only at the points corresponding to the singular-beam vortices, where the degree of polarization reaches the maximum possible (unity) value and the polarization state corresponds to the second (nonzero) component of the combined beam. In this case, the following question arises: can such a partially coherent [10], nonuniformly polarized field be described in the context of singular optics? In particular, can a vector skeleton similar to the set of *C* points and *L* lines of a strictly coherent field be constructed for such a field? It is known [11, 12] that phase singularities can exist not only for the complex amplitude of a coherent monochromatic wave, but also for an arbitrary complex parameter of the field. To solve the outlined problem, a complex parameter of the field should be constructed which includes both ellipsometry characteristics (the polarization azimuth and the ellipticity angle), which are sufficient for describing completely polarized beams, and the degree of polarization.

In this work, we show that the classical concept of the degree of polarization can be generalized to a complex polarization parameter representable in the Stokes space. A new type of polarization singularity, namely, phase singularities of the complex degree of polarization (CDP), is introduced in terms of this parameter, and basic laws governing the formation and evolution of such singularities in partially coherent vector fields are demonstrated. Note that the introduced concept of CDP is not identical to such a two-point polarization parameter of a field as the complex degree of *mutual* polarization [13].

The optical degree of polarization is conventionally introduced as the real nonnegative quantity [8, 9, 14–16]

$$P = \frac{I_p}{I_p + I_u}; \quad 0 \le P \le 1, \tag{1}$$

where I_p and I_u are the intensities of completely polarized and completely unpolarized beam components, respectively, or in terms of the Stokes parameters

$$P = \sqrt{s_1^2 + s_2^2 + s_3^2}, \tag{2}$$

where $s_i = S_i/S_0$, $i = 1, 2, 3$, are the normalized Stokes parameters, $S_0 = I_0 + I_{90}$, $S_1 = I_0 - I_{90}$, $S_2 = I_{+45} - I_{-45}$, and $S_3 = I_r - I_l$. All of the Stokes parameters of a nonuniformly polarized field are functions of the spatial coordinates (x, y). Note that $P = 1$ for a completely polarized beam or at a cross-sectional point of a nonuniformly polarized strictly coherent beam, $0 < P < 1$ for a partially polarized beam, and $P = 0$ for a completely unpolarized beam. The Stokes parameters can be both positive and negative. For example, the normalized second, third, and fourth Stokes parameters for beams with orthogonal polarization states are $\{s_1, s_2, s_3\}$ and $\{-s_1, -s_2, -s_3\}$. Thus, the Stokes parameters contain information on the specific beam polarization state such as the polarization azimuth $\alpha = 0.5\arctan(S_2/S_1) \equiv 0.5\arctan(s_2/s_1)$ $(-\pi/2 \le \alpha < \pi/2)$ and the ellipticity angle $\beta = 0.5\arctan(S_3/S_0) \equiv 0.5\arctan s_3 (-\pi/4 \le \beta \le \pi/4)$. However, this information is lost if the degree of beam polarization is determined in terms of quadrature quantities (2).

The notion of the CDP is based on the ideas developed in [16]. If the beam amplitude is not of interest and only its polarization state is important, the beam can be described in terms of the so-called circular complex polarization variable

$$\chi_{r,l} = (E_r/E_l) = (|E_r|/|E_l|)e^{i(\varphi_r - \varphi_l)}, \tag{3}$$

where E_r and E_l are the components of the circular Jones vector. The quantity $\chi_{r,l}$ is a function of two real variables, the amplitude ratio $|E_r|/|E_l|$ of the right and left circularly polarized beam components and their

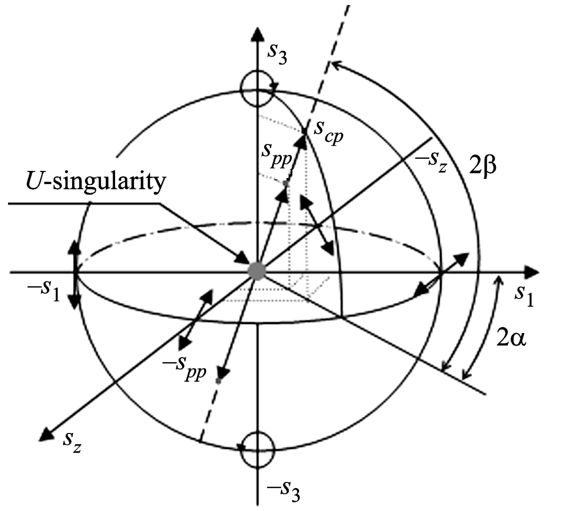

Fig. 1. Stokes-space representation of the completely and partially polarized beams. The Poincaré sphere represents completely polarized fields. The interior of this sphere corresponds to partially polarized fields; the coordinate origin corresponds to an unpolarized field. The vectors $\mathbf{s}_{cp}$ and $\mathbf{s}_{pp}$ describe the polarization vectors of completely ($P = 1$) and partially ($P = 0.64$) polarized fields. The path from $\mathbf{s}_{pp}$ to $-\mathbf{s}_{pp}$ passes through a U singularity.

phase difference $\varphi_r - \varphi_l$. In the proper coordinates of the polarization ellipse, the quantity $\chi_{r,l}$ is written in terms of the polarization azimuth and ellipticity angle as follows:

$$\chi_{r,l} = \tan(\beta + \pi/4)e^{-i2\alpha}. \tag{4}$$

The polarization variable unambiguously determines the polarization state of a completely (in general, elliptically) polarized beam on a circular complex plane [16] and, in the stereographic projection, on the Poincaré sphere of unit radius. The use of the Poincaré sphere is preferred, because it admits the mapping of not only completely polarized beams corresponding to the sphere surface, but also partially polarized beams corresponding to the sphere interior. The origin of the coordinate system corresponds to zero degree of polarization: $s_1 = s_2 = s_3 = 0$ (see Fig. 1). The points outside the Poincaré sphere do not correspond to any polarization state. The use of the CDP defined as

$$\mathcal{P} = P\chi_{r,l}, \tag{5}$$

and the corresponding polarization vector $\mathbf{s} = s_1\mathbf{i} + s_2\mathbf{j} + s_3\mathbf{k}$ ($|\mathbf{s}| = P$) in the Stokes space ensures the description of beams of an arbitrary state and polarization.

Note the significant difference in the motion of a representation point in the Stokes space, which corresponds to the motion in the beam cross section, for strictly and partially coherent fields. The representation point of a coherent field moves only on the sphere surface, so that, e.g., a path connecting a C_r and C_l point

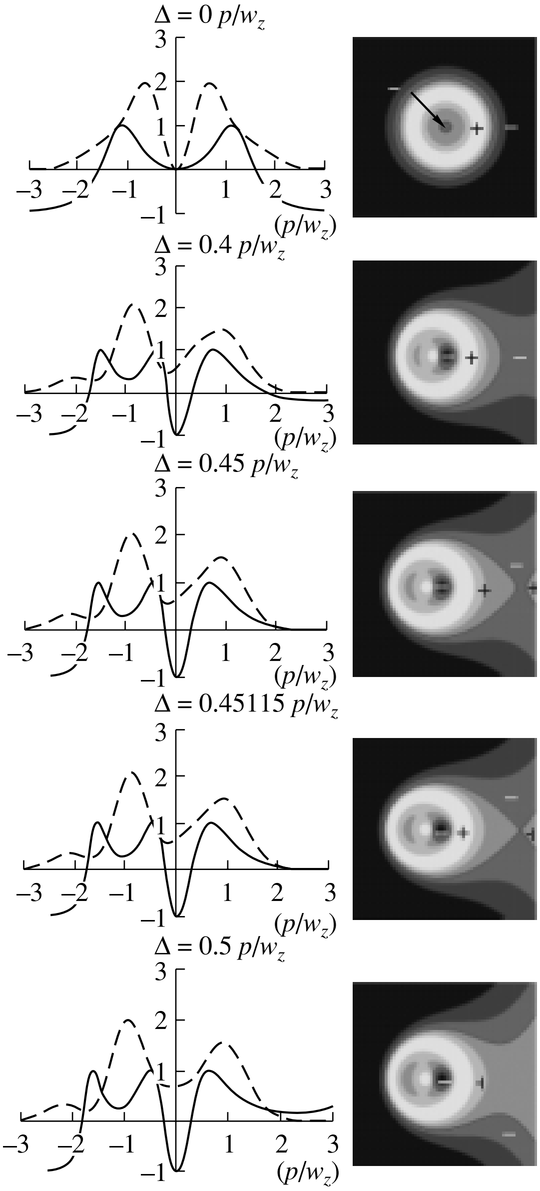

Fig. 2. Left panels: the distributions of the (dashed curve) combined-beam intensity and (solid curve) CDP over the dimensionless parameter ρ/w_z along the direction of the displacement of the LG01- and LG11-mode centers for various values of this displacement Δ. Right panels: U singularities in the beam cross section. The signs ± denote the orthogonal states of partial polarization.

corresponding to the right- and left-circular polarization, respectively, inevitably crosses the large-diameter circle, i.e., the Poincaré-sphere equator (L contour). The representation point of incoherently mixed orthogonally polarized beams moves along the Poincaré-sphere diameter containing the initial point. In this case, the CDP is singular, i.e., $P = 0$ and the polarization state is indeterminate, for the field elements where the intensities of two orthogonally polarized, mutually incoherent beams are equal. The signs of the second, third, and fourth normalized Stokes parameters change simultaneously upon the crossing of such a field element. Note that $0.5\sum_{i=0}^{3} s_i^{(pp)} s_i^{(-pp)} = 0$ for the vectors $\mathbf{s}_{pp}$ and $-\mathbf{s}_{pp}$ (see Fig. 1). This is indicative of the orthogonality of the polarization states [6].

Thus, the Poincaré-sphere center corresponds to specific vector singularities absent in strictly coherent nonuniformly polarized fields. A singularity of such a type can be called the U (*unpolarized*) singularity or CDP phase singularity. The Poincaré-sphere points describing a completely polarized field at the points where the amplitude of one partial beam is zero correspond to the P (*polarized*) singularities. The set of U and P singularities constitute the vector skeleton of a partially coherent combined beam.

The experimental algorithm for constructing such a skeleton involves measuring the coordinate distributions of the intensities corresponding to six polarization states (see Eq. (2)) and plotting the CDP spatial distribution

$$\mathcal{P} = \sqrt{s_1^2 + s_2^2 + s_3^2} \times [\tan(0.5\arcsin s_3 + \pi/4)\exp(-i\arctan s_2/s_1)]. \quad (6)$$

Then, the field elements at which the CDP has the limiting values are determined.

Figure 2 presents the result of the numerical simulation of U singularities for the case of the incoherent superposition of orthogonal LG01 and LG11 modes with a power ratio of 1 : 0.45 [6, 10]. The figure shows the distributions of the intensity and CDP in the cross section of the combined beam in the direction of the controlled displacement Δ between the mode centers (vortices). The distributions are plotted versus the dimensionless radial coordinate ρ/w_z specifying the typical transverse scale of an LG mode. The external U contour on the ring $\rho/w_z \approx 1.5$ corresponds to zero CDP modulus. The inner U contour has the same meaning, but appears for very low CDP values near the ring $\rho/w_z \approx 0.15$ and, therefore, is not identified in the left panels. It is seen that both U contours are deformed only slightly even if the transverse displacement is fairly large (up to $\Delta \approx 0.4\rho/w_z$). With a further increase in the distance between the mode centers, a new U singularity appears from infinity on the right and crosses the external initial contour for some Δ value. However, such U contour crossing is unstable. A further increase in the mode displacement leads to the formation of a new U contour closing at infinity. It is interesting that, as Δ varies in a narrow range from $0.45\rho/w_z$ to $0.5\rho/w_z$, the coordinate distribution of the combined-beam intensity (left) remains almost unchanged, while the CDP undergoes a complete evolution cycle. The ± signs correspond to the cases of the dominant LG01 and LG11 modes, respectively. According to the figure, the following sign rule holds in all cases: the U singulari-

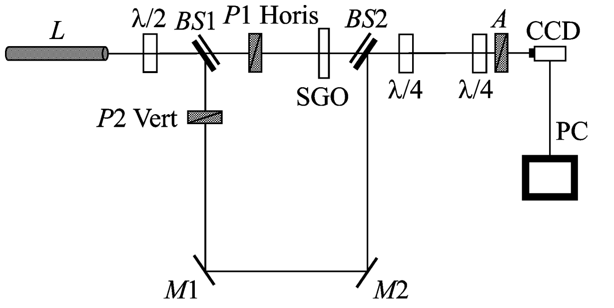

Fig. 3. Scheme of the experiment: *L*, laser; λ/2 and λ/4, half- and quarter-wave plates, respectively; *BS*1 and *BS*2, beam splitters; *P*1 and *P*2, polarizers; *M*1and *M*2, mirrors; *SGO*, singularity-generating object; *A*, analyzer; *CCD*, CCD camera; and *PC*, personal computer.

ties separate regions with orthogonal polarization states.

The scheme of the experimental reconstruction of *U* and *P* singularities and the vector skeleton of partially coherent, nonuniformly polarized combined beams is shown in Fig. 3. A path difference much larger than the laser coherence length is set in the lower arm of the interferometer. The half-wave plate at the interferometer input is used to smoothly vary the arm-intensity ratio for a constant total intensity at the interferometer output. The interferometer arms are equipped with polarizers which form the orthogonal linear polarizations. The quarter-wave plate at the interferometer output makes the mixed-beam polarizations orthogonal elliptical or circular depending on the plate orientation. One interferometer arm includes an object generating a beam with phase singularities. This object is a computer synthesized hologram for the LG-mode reconstruction or a diffuser forming a speckle field. A quarter-wave plate and a linear analyzer in front of a PC-linked CCD camera are used to record the spatial intensity distributions necessary to determine the Stokes parameters and the CDP.

Figure 4 presents the first experimental results on the reconstruction of the vector skeleton of a combined beam formed by the incoherent superposition of a scalar (uniformly polarized) speckle field (Fig. 4a) and a coaxial plane wave with the orthogonal polarization. Figures 4b–4d show fragments of the parametric dynamics of *U* singularities for various ratios $\langle I_S\rangle/I_R$ of the mean speckle-field and the reference-wave intensities. The number, sizes, and shapes of the *U* contours vary with this ratio, while the locations of the *P* points remain the same. As expected, the number and sizes of the *U* contours decrease with the ratio $\langle I_S\rangle/I_R$. The fragments exhibit regions in which the pattern is close to the *U*-contour crossing.

Thus, the concept of CDP introduced in this work makes it possible to describe beams of arbitrary types, states, and degrees of polarization. A new class of optical fields formed due to the incoherent superposition of

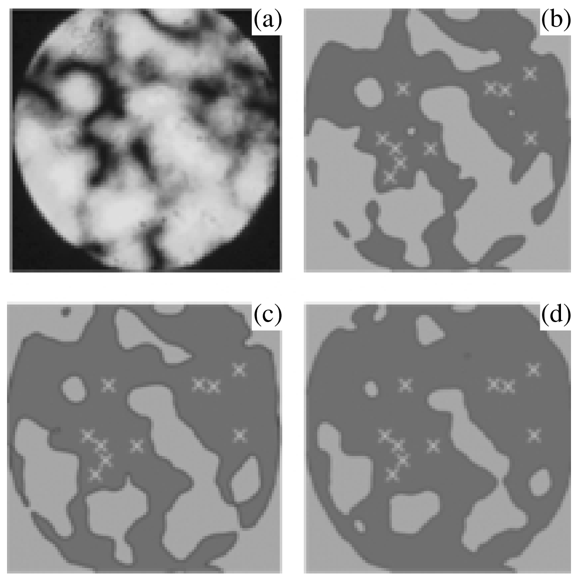

Fig. 4. (a) Studied speckle-field region and (b–d) the fragments of the parametric dynamics of *U* singularities for various intensity ratios $\langle I_S\rangle/I_R$ of the mean speckle field and the orthogonally polarized reference wave: $\langle I_S\rangle/I_R$ = (b) 1, (c) 0.5, and (d) 0.25. The grey scale corresponds to the orthogonal forms of partial polarization. The corresponding areas are separated by the *U* contours. The crosses mark the *P* singularities.

orthogonally polarized beams is analyzed for the first time using the concept of CDP. It is shown that vector singularities called *U* contours and *P* points, which are absent in strictly coherent beams and corresponding to CDP extrema, are formed in such combined beams. Note that the *U* contours can appear due to the incoherent superposition of beams free of phase singularities. On the contrary, the *P* points can appear only if zero-amplitude points exist in at least one beam. *P* singularities are structurally stable and observed for any intensity ratio of the mixed beams, while *U* singularities appear only if the partial-beam intensities are equal on a line at which a CDP phase singularity exists, i.e., the degree of polarization is zero and the polarization state is indeterminate. The CDP extrema determine the specific vector skeleton of a partially coherent, spatially nonuniform polarized field. In the general case of the incoherent mixing of elliptically (orthogonally) polarized beams, the experimental reconstruction of the vector skeleton of the combined beam requires the complete Stokes-polarimetric analysis of the field.

Note that the analysis based on Eq. (3) was performed under the assumption of monochromatic radiation. However, the final result given by Eq. (6) is more general, because the Stokes-parameter definitions do not include the radiation wavelength, the photon energy, and the degree of coherence. The CDP singularities can also be found in the case of the superposition of orthogonally polarized beams with different frequencies, which have phase singularities, as well as polychromatic (white-light) beams. The reconstruction of the vector skeleton of such a field certainly implies a special choice of the polarization elements and detectors.

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