Referenceless testing of vortex optical beams

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A new simple technique for determining the phase handedness (clockwise or counterclockwise) of vortex optical beams, which does not require implementation of an interferometric arrangement, is introduced. It is shown that both the phase handedness and the modulus of an azimuthal mode index of the beam can be directly and unambiguously determined on the basis of bending of interference fringes in a "strip" Young's interference experiment. The initial results are obtained by simulation, partially demonstrated and discussed. Applicability of the proposed technique to testing elementary and complex vortex-bearing optical beams and fields is discussed.

Keywords:

1. Introduction

Intriguing properties of defect-bearing optical beams and fields [1]–[3] as well as the promising applications of such light structures [3]–[5] have attracted considerable attention recently. Investigations in this field constitute an important chapter of modern physics, called by Prof. Soskin singular optics [4], [5]. Vortex optical beams whose wave fronts possess helical structure in the vicinity of points of vanishing amplitude are the typical object of interest of singular optics. The simplest examples of such beams are the Laguerre–Gaussian modes LG_n^m , where the subscript *n* refers to the order of Laguerre polynomial [6], and the superscript *m* refers to the azimuthal mode index or the so-called topological charge of the vortex optical beam. The former characterizes the number of nodes along radial distribution of wave amplitudes at "doughnut" mode, and the later is equal to the value of a phase change in a closed loop around the circumference of the beam axis in 2π measure [4].

While solving many tasks of singular optics, it is necessary test the structure of a vortex optical beam. Particularly, one must be able to determine the handedness (the sign of twirling – clockwise or counterclockwise) of the tested wave front. It is

important for the study of peculiarities of the phase conjugation of vortex beams [7]. behavior of multiple (higher-order) vortices [8], statistics of phase defects in random speckle fields [9], etc. The first experimental technique for obtaining zerograms of the vortex-bearing optical fields has been introduced [9] using an off-axis interferometric arrangement. Positions of the vortices are determined by the typical "forklets" (bifurcations) of interference fringes arising under coherent superposition of the tested field with a tilted plane reference wave. However, it is well known [10], [11] that the handedness of the wave front cannot be determined unambiguously from an off-axis interference experiment, if mutual orientation of the vortex beam and the reference one is unknown beforehand. It is regularly observed by experimentalists in singular optics that a "bright forklet" is replaced by a "dark forklet" and vice versa under gradual change in the phase of the reference wave within a half-period. The wave front handedness may be unambiguously determined only using an on-axis interferometric arrangement, when the vortex beam of interest is mixed co-axially with a plane reference wave or with a divergent wave whose radius of curvature is less than the radius of curvature of the tested beam [4], [11]. Snail-like interference pattern arising in this case provides a direct testing of the wave front twirling. It is clear that interferometric testing of vortex-bearing beams and fields always involves an implementation of a cumbersome optical arrangement needing precise adjustment. Moreover, conventional interferometric techniques occur to be insufficient and even inadequate in the case of testing complex, inhomogeneously spatially polarized but fully coherent optical fields, as well as in the case of partially spatially coherent vortex beams synthesized from the set of mutually incoherent (statistically independent) LG-modes with different n but the same m, like those introduced in papers [12], [13]. Really, in the first case the visibility of interference fringes strongly depends on the states of polarization of the tested field at the running point of the plane of analysis. and of the reference wave; in the second case a reference wave cannot be coherent simultaneously with all mixed statistically independent LG-modes.

In this paper we introduce a new technique for testing the handedness of vortex optical beams, which does not involve implementation of an interferometric arrangement. This technique has been derived by us from the Young–Rubinowicz model of diffraction phenomena [1], [14]–[16], which had been fruitfully applied earlier, partially to solve holographic problems [17]–[26]. Similar approach has been recently used in the observation of an X-ray vortex [27]. The essence of the proposed technique consists in the implementation of a "strip" Young's interference experiment. Namely, one places an opaque strip in front of the tested vortex beam, and observes interference fringes within the geometrical shadow region of the strip, which are produced by the Young's edge diffraction waves (EDWs) [1], [14]–[16]. It will be shown that bending of interference fringes characterizes directly and unambiguously both the handedness of a phase of the tested vortex beam and the modulus of an azimuthal mode index |m| of this beam. Being substantially referenceless, the proposed technique may be applied to testing both elementary complexity.

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2. Background

2.1. Insufficiency of a two-pinhole Young's interference experiment

It is known that the vortex optical beams, a kind of LG-modes, are the beams with a separable phase [12]. It means that the phase factor of the wave field associated with any member of a family of such beams may be represented as a product of two exponential multipliers, one of them depending only on ρ , and the other depending only on ϕ ($\mathbf{\rho} = (\rho, \phi)$ is the position vector of running point of a vortex beam in polar coordinates). To reveal phase information characterizing the helical wave front of a vortex beam avoiding interferometric procedure, one can implement a common Young's interference experiment. Namely, one can use an opaque screen with two small pinholes (whose position vectors are $\mathbf{\rho} = (\rho, \phi)$ and $\mathbf{\rho}' = (\rho', \phi')$) placed in front of the beam of interest and observe interference fringes behind such a screen. Altering spatial separation of the pinholes, one can obtain, in principle, the total phase map of the beam. Particulary, one can determine the sign of twirling of the vortex beam. However, practicability of such procedure seems to be questionable on the following reasons

First, changes in spatial separation of the probing points at the tested beam are accompanied with the changes of spatial frequency and orientation of Young's interference fringes. It results in considerable difficulties in matching the results of partial experiments. Recall in this context that in the famous Thompson's experiment on verification of the Van-Cittert–Zernike theorem [28] only one opaque screen with the fixed separation of two pinholes is used, while all the other parameters of the experiment are altered.

Second, visibility of the Young's interference fringes is directly connected with the coefficient of coherence of the field at two probing points only in the case of spatially homogeneous (in intensity) fields. This condition, being usually fulfilled in classical interference experiments [28], is always violated when one analyzes vortex-bearing beams. On the other hand, visibility of the Young's interference fringes is directly connected with the intensity ratio at the probing points only in the case of a fully coherent tested field. In general case [12], fringe visibility is affected both by spatial coherence and by intensity ratio of the field at the probing points. So, correct interpretation of experimental data requires special precautions.

Third, so-called aperture (pinhole) function always appears in the analysis of the standard two-pinhole Young's interference experiment [16], [29], [30]. Being, in general, loosely defined, this function proves nevertheless to be a critical one in some important cases. Namely, it governs well-known Wolf's diffraction-induced spectral changes in polychromatic radiation undergoing the Young's interference testing [31], [32]. Also, the dependence of interference pattern parameters on the shape and size of pinholes is often observed by experimentalists in coherent optics.

In our opinion, in the case of singular optics the most perfect and adequate arrangement of interference experiment is that realized by Young in 1802 with an opaque strip screen [33], [34].

2.2. Strip Young's interference experiment

The idea of applying a strip Young's interference experiment to test vortex optical beams can be elucidated in the simplest way using the notations of Fig. 1. Assume that an opaque strip of width d is placed in front of $LG_0^{\pm 1}$ mode (doughnut mode of the lowest order), symmetrically in respect to the beam axis with zero amplitude. In accordance with the Young's diffraction paradigm, interference fringes arising within the geometrical shadow region behind the strip result from superposition of the wavelets from edge retransmitters which are thought to be located at each point of two rims of the strip. Let us now predict the expected interference pattern that follows from the "Rubinowicz's representation of the Kirhhoff's diffraction integral" with the principle of stationary phase taken into account [1], [14]–[16].

First of all, the Rubinowicz's formalism leads to peculiar reflection coefficient describing the amplitude and phase structure of the EDW as a function of the diffraction angle θ . For the purpose of this study, it is sufficient to restrict our consideration to the case of very small diffraction angles $\theta \leq 10^{-2}$ rad. Let us additionally take into account the fact that the tested vortex-bearing beam under paraxial propagation is well fitted by quasi-plane-wave approximation. Thus, one can use the simplest form of the polar diagram of the EDW (that becomes precise in the case of a plane probing wave with the wave vector normal to the screen plane) $A(\theta) \sim \cot(\theta/2)$ [15]. Besides, in accordance with the Rubinowicz's reflection coefficient and the principle of stationary phase, each point of smooth screen edge retransmits the probing wave predominantly in the direction perpendicular to the edge's tangent. So, the pattern at the height r (see Fig. 1) is formed mainly by the edge retransmitters (or so-called critical



Fig. 1. Notations for analysis of the strip Young's interference experiment for testing vortex beam: $d \rightarrow d$ the strip width, $\mathbf{\rho} = (\rho, \phi)$ and $\mathbf{\rho'} = (\rho', \phi')$ – the position vectors of the edge retransmitters forming an interference pattern at height *r*.

points of the second kind) with the position vectors $|\mathbf{p}| = |\mathbf{p}'| (\Delta \mathbf{p} = 0)$. The conclusions following directly from the above mentioned behavior of the EDW are listed below:

1. Spatial frequency of interference fringes μ , behind a screen with parallel rims is constant, being determined by the relation among the wave length of the probing radiation λ , screen width d, and the distance between the diffracting screen and observation plane L

$$\mu\approx\frac{d}{\lambda L}\approx\frac{\theta}{\lambda};$$

2. Due to angular amplitude structure of the LG-mode, one expects that the amplitudes of wavelets from the actual edge retransmitters for each r are equal to each other. Besides, owing to very small θ , one can neglect the angular dependence of the EDW amplitude. As a consequence, visibility of the Young's interference fringes is close to unity anywhere behind the strip (though a mean illuminance at the pattern from the LG-mode changes with r).

Let us now determine the phase structure of the interference pattern behind the screen in two distinctive cases.

If an opaque strip is illuminated by a vortexless beam, such as a plane wave of the mode LG_0^0 , then the phase difference of wavelets from the actual edge retransmitters at the center of the geometrical shadow region $(x=0) \Delta \varphi = \varphi - \varphi'$ is equal to zero for each r, irrespective of r and $\Delta \phi = \phi - \phi'$. Thus, one expects to observe uniform straight interference fringes parallel to the strip rims, with the maximum at the center of the diffracting strip. The result of simulation for this case is illustrated in Fig. 2**a**.

More complicated structure of interference pattern is realized in the case when a strip is illuminated by a vortex-bearing beam. Since the phase of a wave associated with such a beam changes by $m2\pi$ in a closed loop around the circumference of the beam axis, one immediately concludes that for m = 1 the edge retransmitters at $r = 0^{-1}$ (at the "equator" of the vortex beam y = 0, $\Delta \phi = \phi - \phi' = \pi$) prove to be out-of-phase by π . It means that the Young's interference fringes at the "equator" of the LG_0^1 mode are shifted by a half-period in respect to the case shown in Fig. 2a. At the same time, interference fringes at "north" and "south" poles of the beam (where $\Delta \phi \rightarrow 0$), are located similarly as in Fig. 2a owing to $\Delta \phi \to 0$ (it is true asymptotically, as $d \to 0$) For that reason, interference fringes at two poles are shifted by a period. This conclusion follows, first of all, from the symmetry considerations. Actually, taking into account rotational symmetry of a vortex-bearing beam, one expects that the structure (bending) of the interference fringes is invariant to the rotation of the coordinates. Also, this conclusion is in agreement with the fact that out-of-phase by π interference fields at "north" and "south" poles of the beam are associated with in-phase interference patterns. The result of simulation of interference fringes produced by a doughnut mode behind a strip, following the evident law

$$\Delta \varphi(d, r) = \pi - \arctan \frac{2r}{d}$$



Fig. 2. Simulated interference patterns arising in a strip Young's experiment: behind the strip illuminated by a homogeneous wave (**a**), behind the strip illuminated by LG_0^{-1} mode (**b**).

is shown in Fig. 2b. For this example we choose $d = 0.4w_z$, where [12]

$$w_z = \left(w^2 + \frac{4z^2}{k^2 w^2}\right)^{1/2},$$

and w being a spot-size at the waist of the beam, z being the distance from the waist of the beam to the diffracting strip, and $k = 2\pi/\lambda$.

The result represented in Fig. 2b shows a simple way for experimental determination of the phase structure of a vortex-bearing optical beam through the strip Young's interference experiment, without using an additional reference wave. In Section 3 we will show experimentally, in particular, that the direction of bending of interference fringes unambiguously characterizes the handedness of the wave front of a vortex beam, corresponding directly to the twirling of a snail-like interference pattern arising in the case of co-axial superposition of the vortex beam of interest with a reference wave. Namely, the result shown in Fig. 2b corresponds to the counterclockwise twirling snail.

2.3. Reference interference pattern (RIP) and spatial phase selection (SPS) techniques

Here we present some simulation results showing additional possibilities of testing vortex optical beams by means of the strip Young's interference experiment.

One can implement a strip Young's experiment with two probing beams, one of them being the tested vortex-bearing beam, and the other being a plane-wave-like beam. Therefore the former beam may be coherent or incoherent with the later one. It is especially important for testing partially spatially coherent vortex beams described in [12], [13]. But, anyway, a possibility to use an independent reference wave reduces considerably the requirements referring to the adjustment precision of an optical arrangement.

An interference pattern produced behind a strip illuminated simultaneously by two mentioned beams is a superposition of partial patterns shown in Fig.2a and b. The result of such superposition shown in Fig. 3a demonstrates a decrease in the visibility of the resulting pattern at the "equator" of the vortex beam, where mismatching of two systems of interference fringes is maximal, being equal to a half-period. Bending of interference fringes from a vortex beam, characterizing a *screw dislocation* of the tested wave front, is transformed now into infinitely extended (non-localized) edge dislocation [2], [3] of the visibility of the resulting pattern. Really, visibility changes its sign by π (fringes are shifted by a half-period) by crossing the line of zero magnitude of this function. In such experiment, one compares phases of two partial interference patterns (intensity distributions) rather than phases of two complex optical fields. Thus, we speak about RIP (Fig. 3a) rather than about reference wave, as in common interferometry (and holography) based on the second-order correlations of the mixed optical fields.

Another possibility consists in the preliminary registration of a reference interference pattern shown in Fig. 2a at any amplitude carrier and subsequent illumination of this carrier with the pattern produced by the vortex beam alone. This procedure corresponds to the multiplication of intensity distributions shown in Fig. 2a and b. The corresponding simulation result is represented in Fig. 3b. Preliminary formed amplitude grating plays the role of SPS for the pattern produced by the tested vortex beam. Namely, only the part of interference distribution from the vortex beam which is matched in phase with transparent areas of the selecting grating passes this grating. In this case (Fig. 3b) bending of interference fringes (Fig. 2b) or vanishing of the visibility function (Fig. 3a) is transformed into the discontinuity of fringes, with a half-period shift of fringes up and down the "equator". Thus, one obtains



Fig. 3. Simulated results of the addition of interference distributions (shown in Figs. 2a and b) \neg RIP technique (a), and of multiplication of interference distributions (shown in Figs. 2a and b) \neg SPS technique (b).

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an edge dislocation of amplitude transmittance of the selecting grating used as the matched spatial filter.

Comparing Fig. 2b with Figs. 3a and b, one can conclude that the direct observation of bending interference fringes behind an opaque screen illuminated by the vortex beam alone provides more information than the results of superposition or multiplying of this distribution with the reference one. Namely, the direction of fringe bending characterizes the sign of the wave front twirling, while the dislocations of visibility or amplitude transmittance of the SPS permit only to check the presence of such twirling. Informativity of the RIP technique and the SPS one becomes evident when these techniques are applied to testing vortex optical beams of more complex structure than $LG_0^{\pm 1}$ mode. Now we show that the mentioned techniques, being applied for the higher-order LG-modes, reveal dependence of the experimental results on a strip width d, which is not evident from the above consideration.

For the purpose of this paper, it is sufficient to consider the case when $LG_0^{\pm 2}$ doughnut mode undergoes the Young's strip interference testing, as well as the RIP technique-based processing and the SPS technique-based one. As |m| = 2, the phase of a wave associated with such a beam changes by 4π in a closed loop around the circumference of the beam axis, and the phase difference of the wavelets from the edger retransmitters at the "equator" of the beam is equal to 2π . It means that bending of the Young's interference fringes behind a strip is doubled in respect to the case shown in



Fig. 4. Simulated interference pattern arising behind the strip illuminated by LG_0^{-2} mode for $d = 0.4w_z$ (**a**), the results of the RIP technique-based processing (**b**) and the SPS technique-based one (**c**).

Fig. 2b. Simulation result for LG_0^{-2} mode and $d = 0.4w_z$ is shown in Fig. 4a. One can see from this figure that interference distribution at the "equator" of the vortex beam occurs to be in-phase with that at the "north" and the "south" poles. But the height r_s , where interference fringes are shifted by a half-period, is loosely identified. The results of the RIP pattern technique-based processing and the SPS technique-based one of the LG_0^{-2} mode (for $d = 0.4w_z$) are shown in Figs. 4 b and c, respectively. Edge dislocations of visibility and amplitude transmittance reveal the presence of two half-period shifts of the Young's interference fringes produced by the tested vortex!

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Fig. 5. Simulated interference pattern arising behind the strip illuminated by LG_0^{-2} mode for $d = 0.8w_z$ (**a**), the results of the RIP technique-based processing (**b**) and the SPS technique-based one (**c**).

for $d = 0.8w_z$. Naturally, the height r, where fringe visibility and amplitude transmittance vanish, depends on the ratio d/w_z , which is proved by the direct computations.

The same technique can be applied to testing of arbitrary *LG*-modes, and the comparison of Figs. 4 and 5 shows that the "resolution" of the beam testing increases with an increase in the ratio d/w_z . Of course, the last conclusion is of importance in the case of higher-order *LG*-modes.

3. Experiment

In this section we describe initial experiments demonstrating applicability of the introduced referenceless technique for testing vortex-bearing optical beams. The experiment described below is restricted by the case of the lowest-order doughnut $LG_0^{\pm 1}$ mode. Experiments with the higher-order LG-modes, as well as with vortex-bearing speckle-fields will be represented elsewhere.

3.1. Experimental arrangement

The basic experimental arrangement is shown in Fig. 6a. Following [35], [36], we apply the computer-generated hologram (CGH) based technique to obtain the vortex optical beam of desirable structure. Laser LG_0^0 mode impinges onto CGH, the zeroth diffraction order of which reproduces LG_0^0 mode, while the ±1st diffraction orders are $LG_0^{\pm 1}$ modes with opposite phase twirling. In one leg of Mach–Zehnden interferometer, formed by two beam-splitting cubes and two mirrors, we select one of two first-order components of the diffracting field, *i.e.*, one of two $LG_0^{\pm 1}$ modes, while in other leg we select the zeroth-order component used as an *auxiliary* reference wave. Desirable components of the diffracting field are selected using aperture stops mounted on micropositioners. Focusing lens at the reference leg of an interferometer plays a two-fold role. Firstly, it is used for precise adjusting of the arrangement into on-axis regime. Namely, one can match two beams at the output beam-splitter of an interferometer with high accuracy by aligning the focused reference wave just with a



Fig. 6. Basic experimental arrangement for testing vortex-bearing optical beam (a); the reduced arrangement for referenceless testing vortex-bearing optical beam (b); L – laser, CGH – computer-generated hologram, BSC – beam-splitting cube, M – mirror, AS – aperture stop, FL – focusing lens, OS – opaque screen, ML – magnifying lens, OP – observation plane.

dark center of the vortex beam. Co-axial superposition of the mixed waves at arbitrary distances behind an interferometer output is then provided by fine adjusting of the output beam-splitter. Secondly, in such a manner one obtains a co-axial reference wave with the radius of curvature less than one of the vortex-bearing beams, as it is required for unambiguous determination of a phase twirling from interferometric data obtained in an on-axis experiment [11]. Further, one places an opaque strip at any distance behind the interferometer output and observes the Young's interference fringes at the removed observation plane. In our experiments, we use a metallic needle with polished surface as an opaque screen. In this case we are sure that the result of vortex testing is invariant in respect to the orientation of the screen toward the beam. Additional magnifying lens is used sometimes to expand observed interference pattern. Besides, one can use varying neutral attenuator in any leg of an interference rometer (not shown in Fig. 6a) for maximization of the visibility of a snail-like interference pattern. Note, that the arrangement shown in Fig. 6a is used only for the comparison of the results obtained in an on-axis interferometric experiment and through referenceless technique for testing vortex-bearing optical beams. An opaque diffraction strip is not used in conventional on-axis experiments, and one observes directly a snail-like interference pattern at the observation plane. The reduced (referenceless) arrangement used by us in further experiments is shown in Fig. 6b.

3.2. Experimental conditions

The experimental conditions realized in our study are as follows. We used a single transverse mode He-Ne laser ($\lambda = 633$ nm, power P = 40 mW) as the source of probing LG_0^0 mode. CGH was computed following [35], [36] to obtain $LG_0^{\pm 1}$ modes at ± 1 st diffraction orders. We used the binary approximation of the computed grating, which underwent photocopying on amplitude carrier (holographic photoplate PFG-01) with following bleaching. Thus, we put high-efficient pure-phase CGH into the probing laser beam. Spatial frequency of the grating is 25 mm⁻¹. Note that one would use the higher-order components of the diffracting field produced by such a grating to test LG-modes with |m| > 1 (m is equal to the number of the diffraction order used). It is known, however, that the higher-order diffractions of a CGH are highly sensitive to violation of the boundary conditions, in our case they are highly sensitive to unavoidable grating imperfections [7]. As a result, the higher-order LG-modes occur to be spatially instable: they are broken into isolated and even pushing apart elementary (with |m| = 1) vortices just behind the CGH [7]. That is why we restrict the experiment described here by testing the vortex-bearing beams generated at ± 1 st diffraction orders only. The waist parameter of the vortex beam at the plane of analysis (at the plane of an opaque strip) is $w_z = 1.3$ mm, while the width of an opaque screen is d = 1 mm; thus $d/w_{\star} \approx 0.77$. Focal distance of the focusing length at the reference leg of an interferometer was 100 mm, and microscopic eyepiece 15^{\times} was used by us as the magnifying lens.

3.3. Experimental results

The main experimental result is shown in Fig. 7. Figures 7**a** and **b** illustrate the result of conventional interferometric testing of vortex-bearing beams with |m| = 1 selected from the symmetrical ±1st diffraction orders of a CGH, with clockwise and counterclockwise handedness, respectively. These snail-like structures are obtained in an interferometric arrangement shown in Fig. 6**a**, in absence of an opaque diffracting strip and using a divergent spherical reference wave co-axial with the tested vortex beam. Figures 7**c** and **d** show the corresponding Young's interference patterns formed behind an opaque strip in absence of a reference wave in an arrangement represented in Fig. 6**a** or, equivalently, in a reduced referenceless arrangement Fig. 6**b**. Correspondence of the direction of bending of the Young's interference "snails" is quite obvious.

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Fig. 7. Interference (**a**, **b**) and referenceless (**c**, **d**) testing of clockwise (**a**, **c**) and counterclockwise (**b**, **d**) vortex beams with |m| = 1.



Fig. 8. Results of the RIP technique-based processing (a) and the SPS technique-based one (b) of the Young's interference pattern for mode LG_0^{-1} .

At last, Figure 8 illustrates the results of the RIP technique-based processing and the SPS technique-based one of the Young's interference pattern for mode LG_0^{-1} . Obviously, experimentally obtained results are in agreement with the predictions made in Sec. 2 obtained from the Young–Rubinowicz model of diffraction phenomena.

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4. Conclusions

Summarizing, a simple referenceless technique for testing vortex-bearing optical beams has been developed here resulting from the Young–Rubinowicz model of diffraction phenomena. The proposed technique provides experimental determination for both the handedness and the modulus of an azimuthal mode index of the tested vortex beam in the simplest, in our knowledge, direct and unambiguous way. Vortexness parameters of the tested beam are determined on the basis of bending of interference fringes resulting from a superposition of the edge diffraction waves within the geometrical shadow region of an opaque strip placed in front of the beam. Additional RIP technique and SPS one have been introduced for testing higher-order vortex-bearing beams.

Apart from the fact that the method does not require an adjustment of an optical arrangement with interferometric accuracy (without loss of unambiguity and accuracy of the testing), an important advantage of the proposed technique consists in the possibility to obtain in one step the total phase map of a vortex beam for arbitrary orientation of a strip opaque screen. While the position of interference fringes does not depend on mutual coherence nor on the states of polarization of the partial beams constituting the combined tested vortex beam, the technique introduced in this paper may be applied to the vortex-bearing beams and fields of arbitrary complexity.

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