Young's diagnostics of spatial coherence phase singularities

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ABSTRACT

We report the feasibilities for revealing and diagnostics of unconventional phase singularities into optical fields, namely, the singularities of spatial coherence functions into partially coherent vortex beams. It is shown that the vortices of the spatial coherence function are comprehensively diagnosed through the strip version of the Thomas Young's interference experiment. Namely, the magnitude of a topological charge and its sign are determined, respectively, by the magnitude and the direction of bending of the Young's interference fringes, which are produced by the edge diffraction waves from the rims of an opaque strip positioned in the vortex beam. Such experiment provides complete data on the azimuthal behavior of a phase of the spatial coherence function. On the other hand, non-localized ring singularities of the spatial coherence function and of the complex degree of coherence occurring in the radial distribution of a phase are detected through conventional Young's interference experiment with two pinholes at an opaque screen. It is remarkable that the last of the mentioned coherence phase singularities takes place, when amplitude zeroes of the field are absent. Instead of this, the modulus of the complex degree of coherence vanishes alone.

Keywords: singular optics, partial coherence, Young's interference experiment

1. INTRODUCTION

This paper is devoted to the revealing and diagnostics of unconventional phase singularities into optical fields, namely, the singularities of spatial coherence functions intrinsic to quasi-monochromatic partially coherent vortex beams. The convenient technique for generating such beams is arranging them by co-axial mixing of the weighed mutually incoherent Laguerre-Gaussian modes[1]. Classical technique for diagnostics of phase singularities, which presumes the use of off-axis[2] or on-axis[3] reference wave, is widely used in coherent singular optics[4], but is not inapplicable in *correlation singular optics*[5] (singular optics of partially coherent and polychromatic fields[6-14]), because the reference wave can not be coherent simultaneously with all mutually incoherent modes constituting the combined partially spatially coherent singular beam.

Here we report the alterative technique for diagnostics of phase singularities, which is based on the Young's interference experiment and, being essentially autocorrelation one, is well adapted for diagnostics of spatial coherence phase singularities. Pre-requisites of the reported approach may be found in[15-18] (look for other relevant publications into periodically updated list[19]). Using such technique, one can reveal both the vortices of the spatial coherence function in the vicinity of zero amplitude points and the ring singularities of the complex degree of coherence, which can exist in case when amplitude zeros are absent both in the combined beam and in any its component.

2. PARTIALLY SPATIALLY COHERENT SINGULAR BEAMS WITH A SEPARABLE PHASE OF A COHERENCE FUNCTION

An instructive example of singular optical beams is the Laguerre-Gaussian modes. It is known that such modes are the singularity-supporting beams with a *separable phase*. Namely, in polar coordinates (ρ, ϕ) the phase of a beam is represented by the product[1,4]

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$$f_l\left(\rho/w_z\right)\exp[im\phi]. \qquad (1)$$

Here the first factor depends on the dimensionless radial variable ρ/w_z, and ϕ is the azimuthal variable. So, the azimuthal and the radial behavior of a phase are represented by separate factors.

The simplest partially spatially coherent singular beam is constructed by co-axial mixing of two weighed statistically independent (mutually incoherent) Laguerre-Gaussian modes LG_n^m with the same topological charges m but with different radial indexes n [16]. For any specific ratio of the mode powers, the radial intensity distribution of the combined beam resembles such distribution into isolated $LG_0^{|1|}$ – mode, see Fig. 1.

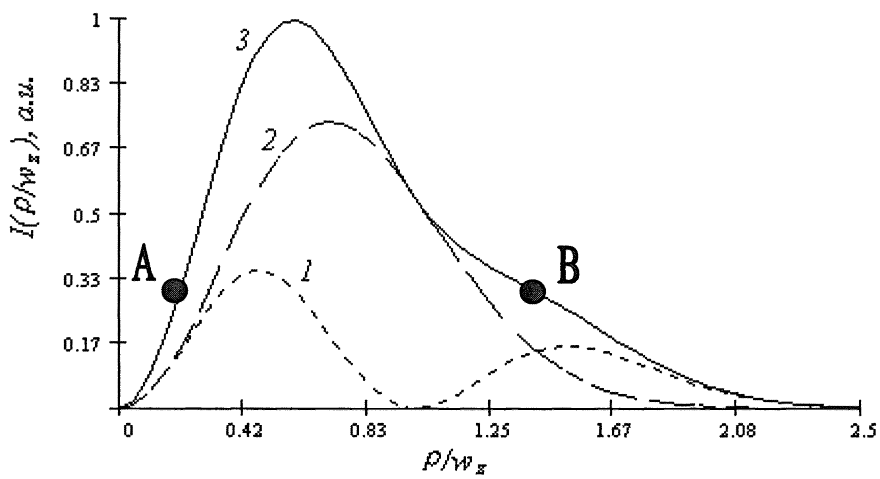


Figure 1. Radial distributions of Laguerre-Gaussian modes $LG_1^{|1|}$ and $LG_0^{|1|}$ (curves 1 and 2, respectively) and of the combined partially coherent beam (curve 3) *vs* dimensionless variable ρ/w_z. Ratio of integral powers of the constituting modes is $P_0/P_1 = 1.45$. A and B are the probing points for revealing the ring phase singularity of the complex degree of coherence.

Partial spatial coherence of such combined beam results from changeable intensity ratio of mutually incoherent constituting modes along the beam radius. Following to the rule for constructing a coherence function, one can see that the phase of the spatial coherence function is also separable in polar coordinates:

$$f_{lj}\left(\frac{\rho_l - \rho'_j}{w_z}\right)\exp[im(\phi - \phi')]. \qquad (2)$$

Thus, one can expect that the phase of the spatial coherence function and the phase of the associated normalized value, namely, the complex degree of coherence, altered at crossing of the radial node of these functions. At the ring where the modulus of the complex degree of coherence vanishes, the phase of this function undergoes singularity.

3. EXPERIMENTAL

The idea of experimental investigation of the azimuthal dependence of the phase of a spatial coherence function at the combined partially coherent singular beam and diagnostics of the vorticity of this function is clear from Fig. 2. An opaque strip of width $2d$ is placed at the tested beam symmertically to its center, and interference fringes arising at the geometrical shadow of the strip are observed. Following to the Young-Rubinowicz model of diffraction phenomena[20-25] we consider these interference fringes as the result of superposition of the *edge diffraction waves*, which are thought as to be re-transmitted by the strip rims. Accounting the stationary phase principle[22, 23], we regard the fringes at any height r as being produced by the edge re-transmitters localized at this height alone.

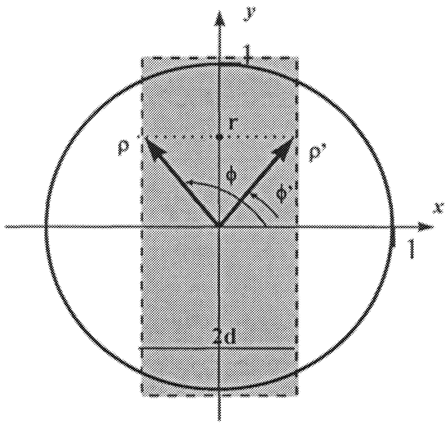


Figure 2. Notations for analysis of the strip Young's interference experiment for testing the azimuthal dependence of the vortex spatial coherence function: $2d$ is the strip width, $\boldsymbol{\rho} = (\rho, \phi)$ and $\boldsymbol{\rho}' = (\rho', \phi')$ are the position vectors of the edge re-transmitters forming an interference pattern at height r.

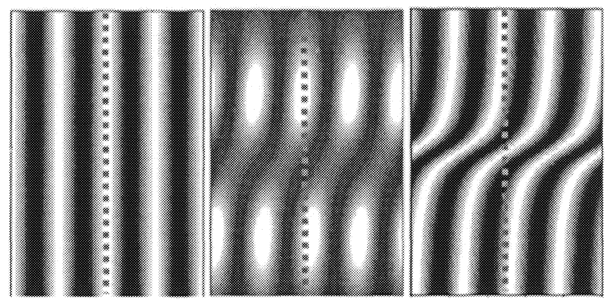


Figure 3. Simulated Young's interference fringes behind an opaque strip illuminated by a vortex-free mode LG_0^0 (**a**), and by the doughnut modes LG_0^{-1} and LG_0^{-2} (**b**) and (**c**), respectively; $d/w_z = 0.4$.

In Fig. 3 one can see the interference patterns observed behind the diffraction strip positioned in front of a vortex-free beam and in front of a single-charged optical vortex. In the first case (a vortex-free beam), see Fig. 3 (**a**), the phases of the edge re-transmitters from both right and left sides are the same. So, one expects to observe the straight Young's interference fringes within the geometrical shadow produced by the superposition of the right- and left-sided wavelets, with the maximum along the mean line of the shadow. But if the tested beam supports the $m-$charged vortex, one observes bended interference fringes with the maximum (if m is even) or with the minimum (if m is odd) at the equator, Fig. 3 (**b**) and 3 (**c**). In general, the phase of the Young's interference fringes for the case under consideration obeys the rule[17]:

$$\Delta\varphi(r, d) = m\left[\pi \pm \arctan\left(r/d\right)\right]. \tag{3}$$

Simplified basic experimental arrangement for creation of the simplest partially spatially coherent vortex beam kind of the one represented in Fig. 1 and for testing the azimuthal dependence of the phase of the spatial coherence function of such beam is shown in Fig. 4. The beam from a laser (LG_0^0 – mode) is splitted into two beams, and an optical path delay is provided in one leg of the mismatched Mach-Zehnder interferometer, which exceeds considerably a coherence length of the used laser. Thus, two partial beams mixed at the output of the interferometer are virtually mutually incoherent.

To create desirable Laguerre-Gaussian modes, we have implemented the computer-generated hologram technique[4]. In two legs of the interferometer we place off-axis computer-generated holograms calculated for reconstruction of the modes $LG_0^{\pm1}$ and $LG_1^{\pm1}$ at the first diffraction orders. Then, at the combined beam at the interferometer output we place a metallic needle as an opaque diffraction screen. An interference pattern is observed within the geometrical shadow region behind the needle.

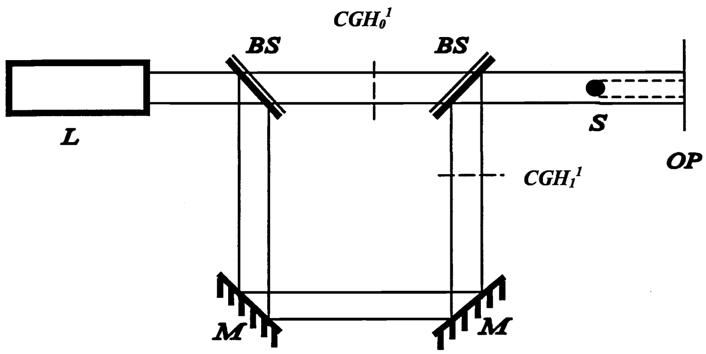
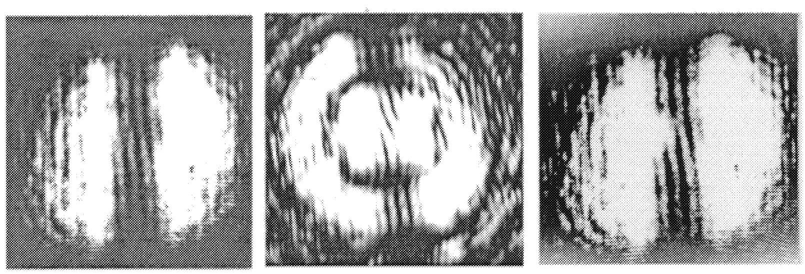


Figure 4. Experimental arrangement for diagnostics of the central vortex of the spatial coherence function of partially coherent singular beam: L – laser, BS – beam-splitters, M – mirrors, CGH_0^1 and CGH_1^1 - computer-generated holograms reconstructing the modes LG_0^1 and LG_1^1, respectively, S – an opaque screen, OP – observation plane.

Fig. 5 demonstrates diffraction diagnostics of the central vortex of the spatial coherence function supported by partially spatially coherent combined beam.



(a) **(b)** **(c)**

Figure 5. Experimental results: diffraction diagnostics of the phase singularities at isolated modes LG_0^1 and LG_1^1, (**a**) and (**b**), respectively, and of the central vortex of the spatial coherence function of the combined partially spatially coherent singular beam (**c**).

So, performing the strip Young's interference experiment, we are in a position to obtain simultaneously complete characteristics of the azimuthal behavior of a phase of the spatial coherence function. Namely, in the single experiment we obtain the phase difference of the wavelets from the opposite sides of the strip from the maximal value of $\phi - \phi'$ at the equator to the minimal one, approaching zero, at the poles of them.

It follows from theoretical consideration that the combined beam resulting from an incoherent superposition of the weighed Laguerre-Gaussian modes $LG_0^{|1|}$ and $LG_1^{|1|}$, beside of the central vortex of the spatial coherence function, supports the ring singularity of the complex degree of coherence, μ_{AB}, which is an analogue of the non-localized dark interference fringe[26] at the isolated mode $LG_1^{|1|}$. Such singularity is detected in Fig. 5 (**b**) by a break and shift of the interference fringes at $\rho / w_z = 1$.

However, it is hardly to reveal such ring singularity of the complex degree of coherence, using the strip Young's experiment. First of all, the amplitude node at the radial intensity distribution of the combined beam is absent, as it is seen from Figs. 1 and 5 (**c**). What is more important, the field of the combined beam of interest does not obey the requirement of statistical homogeneity and isotropy, which is generally accepted in studies of partially coherent fields[27,28]. It means that the statistical moments of the field, including the complex degree of coherence, are dependent on specific choice of the probing points within the beam cross-section. Moreover, the intensity distribution at the beam's cross-section is also nonuniform. As a consequence, the visibility of interference fringes, V, obtained, to say, in two-pinhole Young's experiment is not connected unambiguously with the modulus of the complex degree of coherence being obeying more complicated law:

$$V = \frac{2ab}{a^2 + b^2}\left|\mu_{AB}\right|, \tag{4}$$

where a and b are the amplitudes of the disturbances at the probing points of the beam, A and B, respectively (Fig. 1). One can see that the visibility depends also on the amplitude factor.

In contrast to the diagnostics of essentially two-dimensional azimuthal behavior of a phase of the spatial coherence function considered above, the single two-pinhole Young's experiment occurs to be sufficient for diagnostics of one-dimensional radial behavior of it. In this study we essentially use the classical approach of B. Thompson to determine a phase of the complex degree of coherence in connection with the Van-Cittert – Zernike theorem[28].

Experimental revealing the ring singularity of the complex degree of coherence of the combined beam has been performed using an arrangement of the stellar Michelson interferometer. Namely, a diffraction strip at the interferometer output in Fig. 4 is replaced by a plane opaque screen with two pinholes positioned at the radius of the beam, as it is shown in Fig. 6.

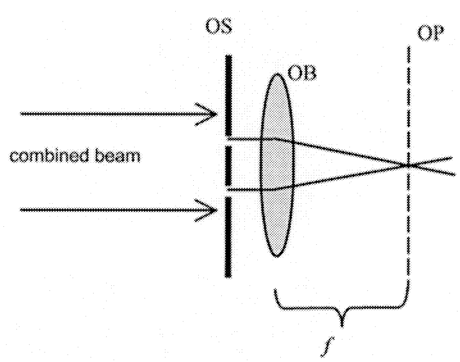


Figure 6. Detecting the ring non-localized singularity of the complex degree of coherence: *OS* – an opaque screen with two pinholes, *OB* – objective, *OP* – observation plane.

Just behind this screen one places an objective and observes an interference pattern at the back focal plane of this objective. The key point in carrying out this experiment consists in proper choice of the probing points at the radius of the combined beam. It has been found, one can specify the probing points, A and B, in such a manner that the intensity of disturbances at such point will be equal to each other both for the combined beam as a whole and for any pairs of the constructing components (two incoherent components for the each pinhole).

Then, the resulting pattern can be considered as superposition of two independent interference patterns from mutually incoherent modes LG_1^1 and LG_0^1. Both partial patterns are of the same spatial frequency owing to the equal interference angles. Besides, both resulting disturbances are of equal intensity. As a result, the amplitude factor in Eq. (4) equals unity, and the visibility of *every* interference pattern is directly connected with the modulus of the complex degree of coherence. Let us emphasize that the field at the focal plane of an objective is *completely* spatially coherent. But, in spirit of[28], we interpret *partial* coherence or *incoherence* of the combined disturbances in points A and B form the magnitude of visibility at the focal plane of an objective.

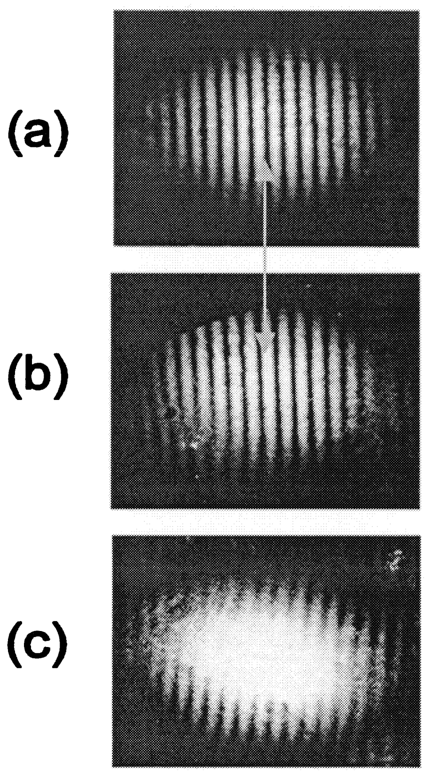


Figure 7. Revealing the ring singularity of the complex degree of coherence of the combined partially spatially coherent singular beam *via* two-pinhole Young's interference experiment: interference fringes of unity visibility produced by the isolated modes LG_1^1 and LG_0^1, (**a**) and (**b**), respectively (arrows show a half-period shift of two interference pattern); a pattern produced by both mutually incoherent modes (**c**), vanishing visibility confirms the presence of the ring phase singularity of the complex degree of coherence at $\rho/w_z \approx 1.45$.

Two partial patterns produced by the isolated modes LG_1^1 and LG_0^1 are shown in Fig. 7 (**a**) and (**b**), respectively. Both patterns are of unity visibility that corresponds to complete coherence of the disturbances of equal intensities produced by the each partial mode. The only difference of two patterns consists in a half-period shift of interference fringes that reflects opposite phases of the complex degree of coherence. So, a dark interference fringe arises at the center of the pattern for the LG_1^1-mode, while the phase of this mode is changed by π at $\rho/w_z = 1$. At the same time, a bright interference fringe arises at the center of the pattern for the LG_0^1-mode. Illuminating an opaque screen with the combined beam, one obtains the result shown in Fig. 7 (**c**). Superposition of two shifted interference patterns of unity visibility and equal average intensity results in vanishing visibility of a pattern. It just means that the modulus of the complex degree of coherence of the combined disturbances at points A and B is equal zero, and the phase of the complex degree of coherence undergoes singularity.

4. CONCLUSIONS

The Young's interference experiment in any its version provides adequate autocorrelation technique for diagnostics of the spatial coherence phase singularities in partially coherent light fields. The key idea of this approach consists in correlation comparison of the disturbances at two different probing points of the tested beam itself, rather than in determining a relative phase difference between the singular beam of interest and a separate reference wave.

Both the azimuthal and the radial behavior of the spatial coherence function are uniquely determined *via* two versions of the Young's interference experiment. The central vortex of the spatial coherence function is diagnosed on bending of interference fringes at the shadow of the diffraction strip, and non-localized ring singularities of the complex degree of coherence are detected on vanishing of the visibility of interference fringes in a two-pinhole Young's experiment.

Generally, the represented technique is applicable for revealing unusual singularities like the ring singularities of the complex degree of coherence, which can exist in case when amplitude zeros are absent both into combined optical beam and into any its component.

REFERENCES

1. S.A. Ponomarenko, "A class of partially coherent beams carrying optical vortices", *Journ. of Opt. Soc. of Amer. A.*, **18**, No. 1. pp. 150-156, 2001.
2. N.B. Baranova, A.V. Mamayev, N.F. Pilipetskii, V.V. Shkunov, B.Ya. Zel'dovich, "Wavefront dislocation: topological limitations for adaptive systems with phase conjugation", *Journ. of Opt. Soc. of Amer.*, **73**, pp. 525-528, 1983.
3. I.V. Basistiy, M.S. Soskin, M.V. Vasnetsov, "Optical wavefront dislocations and their properties", *Opt. Commun.*, **119**, pp. 604-612, 1995.
4. M.S. Soskin, M.V. Vasnetsov, *Singular Optics,* in *Progress in Optics,* E. Wolf, ed., Elsevier, Amsterdam, **42**, pp. 219-276, 2001.
5. P.V. Polyanskii, "Some current views on singular optics", *Proc. SPIE*, **5477**, pp.31-40, 2004.
6. D.M. Palacios, I.D. Maleev, A.S. Marathay, G.A. Swartzlander, "Spatial Correlation Singularity of a Vortex Field", *Phys. Rev. Lett.*, **92** No. 14. - 043905/1-4, 2004.
7. Chien-Chung-Jeng, Ming-Feng-Shih, K. Motzek, Y. Kivshar, "Partially incoherent optical vortices in self-focusing nonlinear media", *Phys. Rev. Lett.,* **92**, No. 4. - 043904/1-4, 2004.
8. M.V. Berry "Coloured phase singularities", *New J. Phys*., **4**, No. 66, pp. 1-16, 2002.
9. J. Leach, M.J. Padgett, "Observation of chromatic effects near a white light vortex", *New J. Phys*. **5**, No. 154, pp. 1-7, 2003.
10. T.D. Visser, E.Wolf, "Spectral anomalies near phase singularities in partially coherent focused wavefields", *J. Opt. A: Pure Appl. Opt.,* **5**, pp. 371-373, 2003.
11. G. Popescu, A. Dogariu, "Spectral anomalies in wave-front dislocations", *Phys. Rev. Lett.*, **88**, 183902, 2002.
12. V.K. Polyanskii, O.V. Angelsky, P.V. Polyanskii, "Scattering-induced spectral changes as a singular optical effect", *Optica Applicata.*, **32**, No. 4, pp. 843-848, 2002.
13. H.F. Schouten, G. Gbur, E. Wolf, "Phase singularities of the coherence function in Young's interference pattern", *Opt. Lett.*, **28**, No. 12, pp. 968-970, 2003.
14. M.S. Soskin, P.V. Polyanskii, O.O. Arkhelyuk, "Computer-synthesized hologram-based rainbow optical vortices", *New J. Phys*., **6**, No. 196, pp. 1-8, 2004.
15. H.V. Bogatyryova, Ch.V. Felde, P.V. Polyanskii, "Referenceless testing of vortex optical beams", *Optica Applicata.*, **33**, No. 4, pp.695-708, 2003.
16. G.V. Bogatyryova, Ch.V.Felde, P.V. Polyanskii, S.A. Ponomarenko, M.S. Soskin, E. Wolf, "Partially coherent vortex beams with a separable phase", *Opt. Lett.,* **28**, pp. 878-880, 2003.
17. Ch.V Felde, "Diffraction diagnostics of phase singularities in optical fields", *Proc. SPIE*, **5477**, pp. 67-76, 2004.
18. Ch.V. Felde, "Young's diagnostics of phase singularities of the spatial coherence function at partially coherent singular beams", *Ukr. J. Phys.*, **49**, No. 5, pp. 473-480, 2004.
19. *http://www.u.arisona.edu/~grovers/SO/so.html.*
20. A. Rubinowicz, "Thomas Young and theory of diffraction", *Nature*, **180**, pp. 160-162, 1957.
21. A. Sommerfeld, Optics, NY: Academic, 1954.
22. K. Miyamoto, E. Wolf, "Generalization of the Maggy-Rubinowicz theory of the boundary diffraction wave", Parts I, II, *Journ. of Opt. Soc. of Amer.*, **52**, No. 6, pp. 615-637, 1962.
23. M. Born, E. Wolf, *Principles of Optics*, Pergamon, New York, 1999.
24. H.V. Bogatyryova, "Properties of Young's holograms", *PhD Thesis*, Chernivtsi, 2000.
25. P.V. Polyanskii, G.V. Bogatiryova, "EDW – edge diffraction wave, edge dislocation wave, or whether *tertio est datur*?", *Proc. SPIE*, **4607**, pp. 109-124, 2001.
26. J. Nye, M.V. Berry, "Dislocations in wave trains" *Proc. Roy. Soc. London,* **A 336**, pp. 165-190, 1974.
27. E. Wolf, B. Thompson, "Two-beam interference with partially coherent light", *Journ. of Opt. Soc. of Amer.*, **47**, No. 10, pp. 895-902, 1957.
28. B. Thompson, "Illustration of the phase change in two-beam interference with partially coherent light", *Journ. of Opt. Soc. of Amer.*, **48**, No. 2, pp. 95-97, 1958.