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Spaceborne linear array imager's spatial resolution for arbitrary viewing angles

Abstract. *Simplified model of image forming in spaceborne linear array sensors at arbitrary sight angles is proposed in this paper. On basis of evaluation of system "lens - linear array detector" modulation transfer function (MTF), the equations were obtained that allow you to determine spatial resolution on Earth's surface. An example of pushbroom imager's MTF determination at sight of Nadir and with different slopes of lens optical axis is given. Image quality changes, which accompany lens optical axis angular inclination were studied. More research needed to determine the impact of lens aberrations on imager's MTF with arbitrary viewing angles.*

Keywords: *remote sensing, spatial resolution, linear array detector, modulation transfer function*

Introduction

Opto-electronic surveillance system (OESS) namely linear array sensors are widely used for Earth observation [1]. Increasing OESS spatial and energy resolution while decreasing the weight, size and power consumption is the pressing problem facing developers of such systems. Modern spaceborne scanners allow you to change the angle of sight, which is determined by the angle between imager's optical axis and Nadir. However, there is significant distortion of the image caused by difference of the angle between the sight axis and the Earth's surface from 90°. Considerable number of articles [2-5] was devoted to quality of the image that is formed by the linear array sensors. Main characteristic that determines image quality and therefore the spatial resolution of OESS is its modulation transfer function (MTF). MTF depends on the aberrations of lens, pixel size and linear array detector (LAD) format, electronic systems, signal processing, and observation conditions. Commonly physical and mathematical models of spaceborne scanners MTF suppose that the plane of the objects (Earth's surface) is parallel to the LAD's plane [6, 7]. It is advisable to explore MTF and spatial resolution of satellite pushbroom sensor when its optical axis is different from the normal to the Earth's surface.

The purpose of the article is to study modulation transfer functions of the satellite pushbroom sensor system "lens – linear array detector" at arbitrary viewing angles and to determine the amount of image distortions of in such OESS.

Image formation model

Physical model of image formation in the LAD OESS is as follows [2]. Solar radiation that is reflected from the Earth's surface passes through the atmosphere and partly gets into lens. The lens forms an image of the object and the background in the LAD's plane. CCD array detector is commonly used as LAD. The LAD converts light distribution into an electrical signal, which, after processing generates a video signal.

In most cases, when modeling the OESS it is considered as invariant linear system. Its one-dimensional MTF $M_s(\nu_x)$ is determined by product of lens MTF $M_L(\nu_x)$ and LAD MTF $M_D(\nu_x)$ as

$$M_s(\nu_x) = M_L(\nu_x) M_D(\nu_x) , \qquad (1)$$

where ν_x is the spatial frequency in the LAD's plane.

High-quality lens MTF may be approximated by [7]

$$M_L(\nu_x) = 1 - 1.22 k_{eff} \lambda \frac{\nu_x}{\eta_{di}} , \qquad (2)$$

where k_{eff} is effective lens aperture and η_{di} is parameter of approximation that defines the difference between real and diffraction limited MTFs for the specific contrast.

Due to finite size $V_D \times W_D$ of LAD's pixel and detector temporal response t_D its MTF is [7]

$$M_D = M_{Ds} M_{Dt} , \qquad (3)$$

where $M_{Ds}(\nu_x) = \mathrm{sinc}(V_D \nu_x)$ is spatial MTF and $M_{Dt} \approx 1$ is temporal MTF. Function M_{Dt} needs more detailed research (because detector time constant can dominate the OESS MTF). But this is beyond the scope of the article. Therefore, we assume that

$$M_D(\nu_x) = \frac{\sin(\pi V_D \nu_x)}{\pi V_D \nu_x} , \qquad (4)$$

where V_D is pixel size in *x*-direction.

Taking into account (2) and (4) OESS MTF is

$$M_s(\nu_x) = \left(1 - 1.22 k_{eff} \lambda \frac{\nu_x}{\eta_{di}}\right) \frac{\sin(\pi V_D \nu_x)}{\pi V_D \nu_x} . \qquad (5)$$

Lens and LAD spatial resolution balancing can be received according to two criteria [4]:

1. The equality of the lens and LAD MTFs at a certain spatial frequency ν_{x1}. Most often it is supposed that $M_L = M_D = 0.5$ (Figure 1). Then

$$M_L(\nu_{x1}) = 1 - 1.22 k_{eff} \lambda \frac{\nu_{x1}}{\eta_{di}} = \frac{1}{2} ;$$
$$M_D(\nu_x) = \frac{\sin(\pi V_D \nu_{x1})}{\pi V_D \nu_{x1}} = \frac{1}{2} . \qquad (6)$$

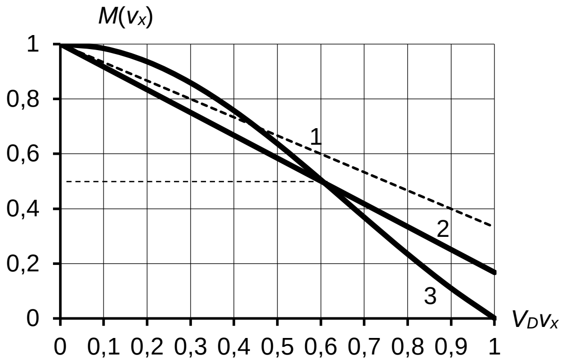

Fig. 1. MTF: 1 - diffraction limited lens, 2 - lens with $\eta_{di} = 0.8$ which is matched to LDA and 3 - LDA with V_D pixel size

From the system of equations (6) matched spatial frequency becomes

(7) $$\nu_{x1} = \frac{\eta_{di}}{2.44 k_{eff} \lambda}\,;\;\; \nu_{x1} = \frac{0.6}{V_D}.$$

Comparing last two equations we find the formula for matching parameters of the lens and LDA

(8) $$k_{eff} = \frac{f_L^{'}}{D_p} = 0.68\frac{\eta_{di} V_D}{\lambda},$$

were $f_L^{'}$ and D_p are rear focal length and diameter of the lens entrance pupil, respectively.

The resolution element of the system "lens – LDA" in detector plane is modeled as rectangle with $V_s \times W_s$ size. Its one-dimensional MTF according to (4) is

(9) $$M_s^a(\nu_x) = \frac{\sin(\pi V_s \nu_x)}{\pi V_s \nu_x},$$

where V_s is resolution element size in *x*-direction.

From the matching conditions of functions (5) and (9) at the level of 0.5 we get system of equations

(10) $$M_s(\nu_x) = 0.5\,;\;\; M_s^a(\nu_x) = 0.5.$$

From the second equation (10) similarly to (7) we have

(11) $$\nu_{x2} = \frac{0.6}{V_s}.$$

Then the first equation (10) considering (8) becomes

$$\left(1 - \frac{V_D}{2V_s}\right)\frac{\sin\left(\pi V_D \dfrac{0.6}{V_s}\right)}{\pi V_D \dfrac{0.6}{V_s}} = \frac{1}{2}.$$

The solution to this transcendental equation is

(12) $$V_s = 1.49 V_D.$$

2. The equality of the lens and LAD MTFs at Nyquist frequency $\nu_N = 1/2V_D$, that is $M_L(\nu_N) = M_D(\nu_N)$.

Using equations (2) and (4) we have

(13) $$1 - 0.61\frac{k_{eff}\lambda}{V_D \eta_{di}} = \frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi}.$$

With equations (13) we get requirements for the lens according to this criterion

(14) $$\frac{D_p}{f_L^{'}}\eta_{di} = 1.68\frac{\lambda}{V_D}.$$

Analysis of equations (8) and (14) for matching lens and detector resolution according to the two considered criteria shows:

1. The OESS spatial resolution as consistent with first criterion is

(15) $$\nu_{res1} = \frac{0.6}{V_D},$$

and according to the second criterion

(16) $$\nu_{res2} = \frac{0.5}{V_D}.$$

2. Contrast of the image according to the first criterion is 0.25, and according to the second criterion is 0.41.

3. For both criteria parameter $\frac{D_p}{f_L^{'}}\eta_{di}$ of the lens depends on inverse of normalized detector pixel size $\frac{V_D}{\lambda}$.

LDA MTF (4) is calculated for the case when OESS optical axis is perpendicular to the Earth surface or object plane and lens focal plane are parallel to each other and perpendicular to the optical axis. When deviation of the optical axis from Nadir occures ("pushbroom" scanning mode) the object plane and the optical axis of the scanner forms an angle of sight θ_v. It leads to degradation of image and increases size of spatial element on the Earth's surface (Figure 2). On the edge of field of view the size $\delta V \times \delta W$ of the resolution element on the Earth's surface increases.

Let's consider the one-dimensional resolution element size with angularity deviation of optical axis from Nadir θ_{vx} along axis x (Figure 2, a). LDA MTF is determined by equation (4), which for arbitrary sight angles would be

(17) $$M_D(\nu_x) = \frac{\sin(\pi V_{Dv} \nu_x)}{\pi V_{Dv} \nu_x},$$

where V_{Dv} is OESS resolution element size in back focal plane of ideal lens. It is determined by projection $\delta V = V_D^{'}$ of LDA pixel on the surface of the Earth.

Let's determine the resolution element size of on the Earth's surface with optical axis angularity deviation θ_{vx} from Nadir in perpendicular to satellite flight direction (Figure 2, a).

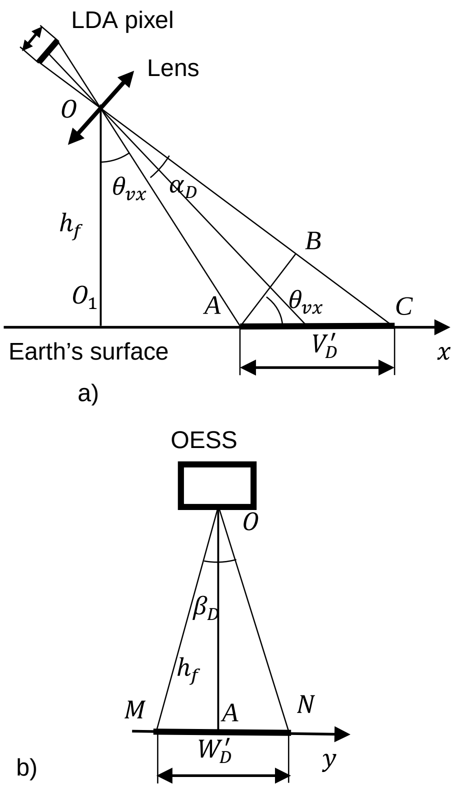

Fig. 2. OESS spatial resolution on the Earth's surface with angularity deviation θ_{vx} of optical axis from Nadir along the axis x: a) - resolution element along axis x and b) - resolution element along axis y

We obtain size $\delta V = V_D^{'}$ of resolution element (which is projection of LDA pixel on the Earth's surface) along axis x from triangle ABC.

(18) $$\delta V = V_D^{'} = \frac{AB}{\cos\theta_{vx}}.$$

Line segment AB according to triangle ABO is

(19) $$AB = OA \sin\alpha_D \approx OA \cdot \alpha_D,$$

and segment OA according to triangle OO_1A

(20) $$OA = \frac{OO_1}{\cos\theta_v},$$

where $OO_1 = h_f$ is altitude of the satellite and α_D is pixel angular size.

After substituting (19) and (20) into (18) we obtain resolution element size at an angle of sight θ_{vx}

(21) $$\delta V = V_D^{'} = \frac{h_f \alpha_D}{\cos^2\theta_{vx}} = \frac{h_f V_D}{f_L^{'} \cos^2\theta_{vx}}.$$

We get size of this element $\delta W = W_D^{'}$ along the axis y from Figure 2, b. From triangle OMN we have

(22) $$\delta W = W_D^{'} = OA \cdot \beta_D = \frac{W_D h_f}{f_L^{'} \cos\theta_{vx}}.$$

Similar ratios can be obtained when sight line deviation from Nadir along the axis y is θ_{vy}

(23) $$\delta V = V_D^{'} = \frac{h_f V_D}{f_L^{'} \cos\theta_{vy}}, \quad \delta W = W_D^{'} = \frac{W_D h_f}{f_L^{'} \cos^2\theta_{vy}}.$$

The difference of the resolution element's opposite sides in case of deviation of optical axis at angles $\theta_{vx} \neq 0$ and $\theta_{vy} = 0$ can be estimated from equation (21). If the first side of the resolution element is determined by (21), then increasing the sight angle on value $d\theta_{vx} = \alpha_D$ will increase the opposite side by

(24) $$d(\delta V) = \frac{2 h_f V_D \sin\theta_{vx}}{f_L^{'} \cos^3\theta_{vx}} d\theta_{vx}.$$

The relative increase in the opposite side is

(25) $$\frac{d(\delta V)}{\delta V} = 2\alpha_D \operatorname{tg}\theta_{vx}.$$

For sight angle $\theta_{vx} = 35^\circ$ and instantaneous field of view $\alpha_D = 8$ μrad the relative increase in the opposite side is equal to $9 \cdot 10^{-3}$, that is $d(\delta V) << \delta V$. This means that the resolution element of the OESS on the Earth's surface can be defined in the form of a rectangle $\delta V \times \delta W$ which sides are determined by equations (20) and (21). Then the area of the resolution element is equal to

(26) $$\delta S = \frac{h_f V_D}{f_L^{'} \cos^2\theta_{vx}} \frac{W_D h_f}{f_L^{'} \cos\theta_{vx}} = \left(\frac{h_f}{f_L^{'}}\right)^2 \frac{S_D}{\cos^3\theta_{vx}},$$

where $S_D = V_D W_D$ is LDA pixel area.

In linear array sensors image scanning at a second coordinate is realized by own motion of the satellite. During line scan period t_l or integration time t_i in TDI detectors image line on the Earth's surface will shift by

(27) $$dy = v_f t_l,$$

where v_f is subsatellite point speed on the Earth's surface (for instance 6.8 km/sec).

The value dy can be considered as spatial resolution along the satellite path. It can be changed by changing scanning frequency f_d or integration time t_i. It should be remembered that increasing frequency f_d reduces storage time in CCD and therefore decreases energy resolution of OESS.

Element resolution reduced size from Figure 2 is determined as

(28) $$V_{Dv} = V_{Dv}^{'} \frac{f_L^{'} \cos\theta_{vx}}{h_f} = \frac{h_f V_D}{f_L^{'} \cos^2\theta_{vx}} \frac{f_L^{'} \cos\theta_{vx}}{h_f} = \frac{V_D}{\cos\theta_{vx}}.$$

Then LAD MTF for arbitrary sight angles is

(29) $$M_{Dv}(\nu_x) = \frac{\sin\left(\pi \frac{V_D \nu_x}{\cos\theta_{vx}}\right)}{\pi \frac{V_D \nu_x}{\cos\theta_{vx}}}.$$

Results and discussion

As an example of the proposed MTF model application let's consider linear array imager with following parameters:

- lens: focal length $f_L^{'} = 850$ mm, entrance pupil diameter $D_p = 200$ mm, spectral range λ_1-$\lambda_2 = 0.5 - 0.76$ μm, spatial resolution is limited by diffraction;

• detector – silicon CCD array (CCD 151): number of pixels $N_D = 5000$, pixel size $V_D \times W_D = 7 \times 7$ μm²; scanning frequency $f_d = 5$ MHz.

Imager is mounted on a satellite with a height of orbit $h_f = 680$ km. Imager sight angles relative to Nadir are: $\theta_{vx} = \pm 35^\circ$ across satellite's path and $\theta_{vy} = \pm 25^\circ$ along the path.

Calculations of imager's MTF will be executed in the following sequence:

1. Imager's angular field of view is evaluated according to equation (17) in respect that the array size is $l_D = V_D N_D = 7 \cdot 10^{-3} \cdot 5000 = 35$ mm. Then

$$2\omega_o = 2\omega_{ox} = V_D N_D = 2\mathrm{arctg}\frac{35}{1700} = 0.041\,\mathrm{rad} = 2.4^\circ$$

.

2. The resolution element size on the Earth's surface will be defined in the following way

2.1. When scanning along the axis x equations (4) and (5) yield

$$\delta V = \frac{h_f V_D}{f_L' \cos^2\theta_{vx}} = \frac{680 \cdot 10^3 \cdot 7 \cdot 10^{-6}}{850 \cdot 10^{-3}} \frac{1}{\cos^2\theta_{vx}} = 5.6 \frac{1}{\cos^2\theta_{vx}}$$

m;

$$\delta W = \frac{W_D h_f}{f_L' \cos\theta_{vx}} = \frac{680 \cdot 10^3 \cdot 7 \cdot 10^{-6}}{850 \cdot 10^{-3}} \frac{1}{\cos\theta_{vx}} = 5.6 \frac{1}{\cos\theta_{vx}}$$

deg.

Figure 3 shows graphs of resolution element size $\delta V \times \delta W$ as a function at sight angle θ_{vx}. Analysis of the results shows that

a) Resolution element has shape of a square with sides of 5.6 m at Nadir;

b) On the edge of the imager's field of view when $\omega_o = 1.2^\circ$ and sight angle is $\theta_{vx} = 0^\circ$, resolution element approximately has shape of a rectangle of size 5.61×5.6 m²;

c) On the imager's optical axis when $\omega_o = 0^\circ$ and maximum sight angle is $\theta_{vx} = 35^\circ$ resolution element approximately has shape of a rectangle of size 8.3×6.8 m².

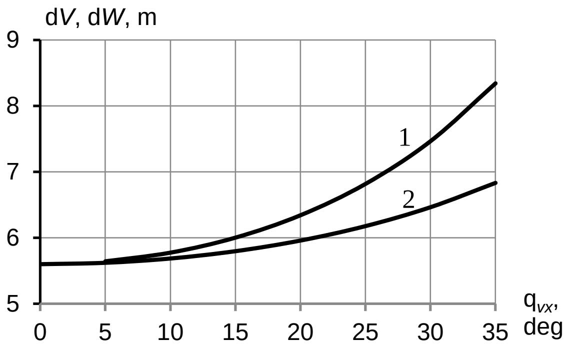

Fig. 3. Resolution element size $\delta V \times \delta W$ on the Earth's surface in center of field of view: 1) - across the flight and 2) - along the flight

Similar results can be obtained with optical axis deviation from Nadir to angle θ_{vy} along the satellite's path:

a) Resolution element has shape of a square with sides of 5.6 m at Nadir;

b) On the edge of the imager's field of view when $\omega_o = 1.2^\circ$ and sight angle is $\theta_{vy} = 0^\circ$, resolution element approximately has shape of a rectangle of size 5.61×5.6 m²;

c) On the imager's optical axis when $\omega_o = 0^\circ$ and maximum sight angle is $\theta_{vy} = 25^\circ$ resolution element approximately has shape of a rectangle of size 6.2×6.8 m².

3. Considering the satellite motion with speed $v_f = 6.8$ km/sec, element size δW along coordinate y will increase by an amount $\Delta W = v_f t_f$,

where t_f is period of signal reading in LDA

$$t_f = \frac{N_d}{f_d} = \frac{5000}{5 \cdot 10^6} = 10^{-3} \text{ sec.}$$

Then $\Delta W = 6.8 \cdot 10^3 \cdot 10^{-3} = 6.8$ m, which significantly degrades imager's spatial resolution.

4. To determine system "lens – LDA" MTF depending on the sight angle θ_{vx} we will use equation (29).

For our example we have $k_{eff} = f_L' / D_p = 4.25$, $\lambda = 0.6$ μm, $\eta_{di} = 1$, $V_D = 7$ μm, $\theta_{vx} = 35^\circ$. Figure 4 shows imager's MTF graphs in Nadir and at maximum sight angle $\theta_{vx} = 35^\circ$.

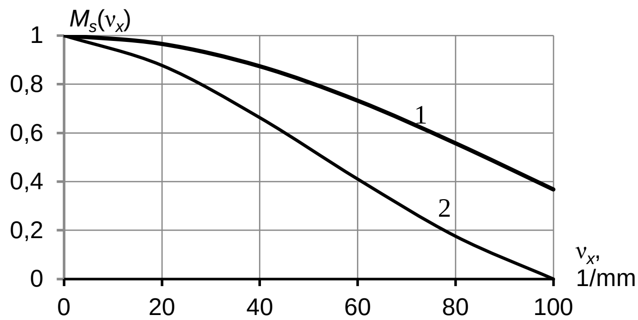

Fig. 4. Imager's MTF when sight angles are 1) - $\theta_{vx} = 0^\circ$ and 2) - $\theta_{vx} = 35^\circ$

5. On Nyquist frequency $\nu_N = 1/2V_D = 71\,\mathrm{мм}^{-1}$ image contrast is approximately: 61% in Nadir and 29% when sight angle is $\theta_{vx} = 35^\circ$. That is, image quality deteriorates by half.

Conclusions

1. Virtually there are no works, which examine changes in linear array imager's MTF when optical axis deviates from vertical direction and which leads to deterioration of image quality at large viewing angles.

2. The proposed physical-mathematical model of linear array imager allowed to determine modulation

transfer function of system "lens - detector" and to formulate two criteria of lens and detector balancing: 1 - lens and detector MTFs equality for certain spatial frequency; 2 - lens and detector MTFs equality for Nyquist frequency.

3. The study of these criterions showed that spatial resolution is $0.6 / V_D$ to the first criterion and is $0.5 / V_D$ to the second criterion, where V_D is size of the array pixel. At the same time, the contrast of the image according to the first criterion is 0.25 and is 0.41 according to the second criterion.

4. The obtained "lens – detector" MTF at arbitrary viewing angles θ_{vx} allows to calculate the resolution element size on the Earth's surface depending on the angle θ_{vx}. It is proportional $\cos^{-2}\theta_{vx}$ across the satellite flight and is proportional $\cos^{-1}\theta_{vx}$ along the flight.

5. Evaluation method for imager's spatial resolution with specified lens and detector parameters was implemented on basis of the MTF. It's analysis showed that:

5.1. Resolution element size on the Earth's surface substantially depends on the sight angle: at Nadir it has shape of a square with sides of 5.6 m, and when the sight angle is $\theta_{vx} = 35^\circ$ it has shape of a rectangle of size 8.3×6.8 m².

5.2. The imager's modulation transfer function is determined by the product of lens and detector's MTFs. Pixel size is considered as projection of the resolution element on the Earth's surface, provided to rear focal plane of the lens. In this case, given pixel size depends on sight angle in accordance with the equations (9) – (11). For example, when changing the sight angle θ_{vx} across satellite path, the pixel size is proportional to $\cos^{-2}\theta_{vx}$ in the same direction and is proportional to $\cos^{-1}\theta_{vx}$ along the path.

5.3. With increasing sight angle imager's MTF narrows, which leads to deterioration of image quality. For example, when changing the sight angle θ_{vx} from 0° to 35° MTF on the Nyquist frequency decreases from 0.61 to 0.29, which significantly impairs the image quality.

6. It is expedient to determine the impact of lens aberrations on imager's MTF with arbitrary viewing angles in further studies.

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REFERENCES

[1] Jensen J.R. Remote Sensing of the Environment: An Earth Resource Perspective. Prentice-Hall, Inc.: Upper Saddle River, NJ., (2000).
[2] Kolobrodov V.G., Liholit N.I. Development of thermovision and television systems. Kiev, NTUU "KPI", (2007).
[3] Holst G.C. Electro-Optical Imaging System Performance, Fifth Edition. JCD Publishing, (2008).
[4] Kolobrodov V.G. Effect of lens aberrations at spatial separation scanner. *Science news of NTUU "KPI"*. 5 (2000), 110 –112.
[5] Vollmerhausen R.H., Reago D., Driggers R.G. Analysis and evaluation of sampled imaging systems. SPIE Press, (2010).
[6] Chrzanowski K. Testing thermal imager. Warsaw: Military University of Technology, (2010).
[7] Schuster N., Kolobrodov V.G. Infrarotthermographie (Zweite, uberarbeitete und erweiterte Ausgabe). Berlin: Wiley-VCH Verlag Gmb & Co. KGaA, (2004).